Ecole Doctorale Carnot-Pasteur

Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :

Équations de Hamilton–Jacobi pour le contrôle robuste à base de noyaux et l'apprentissage par renforcement

Intitulé en anglais du sujet de these proposé :

Hamilton–Jacobi Equations for Robust Kernel-Based Control and Reinforcement Learning

Unité de recherche : IMB (UMR 5584, Université Bourgogne Europe & CNRS)

Nom, prénom et courriel du directeur (et co-directeur) de thèse :

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Domaine scientifique principal de la thèse :

Applied Mathematics — Partial Differential Equations and Optimal Control
Mathématiques appliquées — Équations aux dérivées partielles et contrôle optimal

Domaine scientifique secondaire de la thèse :

Machine Learning — Kernel Methods and Reinforcement Learning
Apprentissage automatique — Méthodes à noyaux et apprentissage par renforcement

<u>Description du projet scientifique :</u>

Gaussian-process regression is a canonical nonparametric tool for **learning dynamics from data**; it produces a posterior mean and variance, and hence confidence regions encoding **epistemic uncertainty**. In model-based RL, uncertainty quantification is central in low-data regimes. From a control viewpoint, epistemic uncertainty naturally motivates robust formulations in which the dynamics is not a single vector field but a family of plausible models; this leads to **minimax dynamic programming** and HJI equations.

Kernel-based learning and **Reproducing Kernel Hilbert Space** (RKHS) representations often express functions as kernel integral operators or inner products with representers. This suggests dynamics where the velocity field is generated by a kernel acting on a distributed control. The resulting control problems are intrinsically **nonlocal** and lead to **integro-HJ equations**, requiring nonlocal viscosity techniques.

This project adopts an analytical approach to bridging continuous-time optimal control/ re-inforcement learning (RL) and **viscosity solutions** of **Hamilton–Jacobi–Bellman/Isaacs** (HJB/HJI) equations. The focus is on two structural features naturally produced by kernel-based learning:
• **Epistemic uncertainty** (e.g. Gaussian-process confidence regions) leading to **differential inclusions** and **minimax** (robust) Hamilton-Jacobi-Isaacs equations;
• **Kernel-induced nonlocal dynamics** (integral-operator / RKHS representations) leading to **nonlocal** (integro-)HJB/HJI equations.

The thesis is organized into four steps.

<u>Step 1</u>: **Viscosity toolkit for robust and nonlocal Hamilton–Jacobi equations.** Establish a reusable analytical framework for well-posedness (comparison/existence/uniqueness), stability, and parameter dependence for (i) robust HJI equations with set-valued dynamics and minimax Hamiltonians, and (ii) nonlocal integro-HJ equations driven by kernel integral operators. Emphasis is on unbounded domains, controlled growth conditions, and stability under perturbations of uncertainty sets and kernels.

<u>Step 2</u>: **Robust control from gaussian process epistemic uncertainty.** Model the learned dynamics as a family of plausible vector fields generated by GP posterior confidence sets. Derive the robust value function, establish dynamic programming, and prove viscosity well-posedness for the associated HJI equation. Study stability as the confidence radius shrinks, yielding a rigorous robust-to-nominal limit.

<u>Step 3</u>: **Kernel-induced nonlocal control dynamics and localization limits.** This step studies optimal control problems with kernel-induced nonlocal dynamics arising from distributed controls in functional spaces. The associated Hamilton–Jacobi–Bellman and Hamilton–Jacobi–Isaacs equations feature nonlocal Hamiltonians. Viscosity well-posedness is established under suitable structural assumptions on the kernel and cost. Localization regimes are then analyzed, showing convergence of nonlocal value functions and equations to their classical local counterparts.

<u>Step 4</u>: **Unified robust+nonlocal models and RL interpretation**. Combine Steps 2–3 into robust nonlocal control models (uncertain kernels / uncertain induced vector fields). Establish stability under joint limits (uncertainty ↓ and localization ↓), and interpret robust model-based policy improvement/model learning (in the spirit of GP-based methods such as PILCO) as consistent approximation schemes for the limiting HJB/HJI equations.

Connaissances et compétences requises :

A PhD student working on robust and nonlocal Hamilton–Jacobi theory for kernel-based RL should have a solid background in: (i) optimal control and dynamic programming; (ii) PDEs and viscosity solutions; (iii) functional analysis (RKHS basics helpful); and (iv) probability at the level needed for interpreting GP confidence sets. Deep RL engineering is not required.