

École Doctorale Carnot-Pasteur

Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :

Géométrie et combinatoire des groupes de trickle

Intitulé en anglais du sujet de these proposé :

Geometry and combinatorics of trickle groups

Unité de recherche : IMB (UMR 5584, Université de Bourgogne & CNRS)

Nom, prénom et courriel du directeur (et co-directeur) de thèse :

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Domaine scientifique principal de la thèse :

Mathématiques

Domaine scientifique secondaire de la thèse :

Description du projet scientifique :

There are numerous groups in the literature defined by relations of the form $xy = zx$ and $x^t = 1$. Prominent examples include right-angled Artin groups, right-angled Coxeter groups, and more generally, graph products of cyclic groups. The present project concerns a specific family of such groups, called *trickle groups*, introduced in a recent preprint by Bellingeri, Godelle and Paris.

Cactus groups are emblematic examples of trickle groups. These first appeared as quasi-braid groups in the study of the mosaic operads and they were subsequently generalized to all Coxeter groups. Their significance was further highlighted in their connection to coboundary categories, mirroring the role of braid groups in braided categories. Additionally, cactus groups and their generalizations to Coxeter groups have found applications in representation theory under various guises.

Trickle groups include other groups naturally related to cactus groups, such as the "Artin" versions of cactus groups (associated with Coxeter groups), which are preGarside groups in the sense of Godelle and Paris. Another example of a trickle group related to cactus groups is a finite index subgroup of the virtual cactus group. Numerous other trickle groups exist within the literature. Among these, there is one originating from dynamical systems and another from knot theory. The first one is the celebrated Thompson group F , introduced by Richard Thompson in 1965 in an unpublished manuscript, which is a group of homeomorphisms of the real line with many unusual properties. The second example are the so-called ordered quandle groups whose presentations come from ordered quandles.

Trickle groups appear to be a reasonable generalization of graph products of cyclic groups, and consequently, of right-angled Artin groups and right-angled Coxeter groups. Therefore, it is natural to explore whether results known for some or all graph products of cyclic groups can be extended to some or all trickle groups.

Relevant questions on trickle groups include:

- Are trickle groups automatic or bi-automatic?
- Which trickle groups admit geometric actions on $CAT(0)$ cube complexes?
- Are trickle groups residually finite? Are preGarside trickle groups residually nilpotent without torsion? Can we determine the Lie algebra associated with their lower central series?
- Which preGarside trickle groups are orderable, bi-orderable, or admit isolated orders?

Additionally, it would be valuable to discover new examples of trickle groups, particularly those arising from areas of mathematics beyond group theory.

The aim of the thesis project is to study the questions posed above. Answers to these questions are well-known for graph products of cyclic groups, and it is possible, even probable, that some of the techniques used for graph products of cyclic groups can be adapted with some effort to trickle groups.

Connaissances et compétences requises :

The candidate must possess a solid foundation in algebra, particularly in combinatorial and geometric group theory. She must be familiar with the notions of free groups, group presentations, group actions, torsion, and the word problem. Familiarity with some concepts in geometric group theory, such as actions on $CAT(0)$ spaces, small cancellation theory, or hyperbolic groups, would be advantageous but is not a prerequisite.

La candidate doit avoir de solides bases en algèbre, et plus particulièrement en combinatoire et géométrie des groupes. Elle doit être familiarisée avec les notions de groupe libre, de présentation de groupe, d'action de groupe, de torsion et de problème du mot. Quelques connaissances en géométrie des groupes, telles que les actions sur des espaces $CAT(0)$, la petite simplification ou les groupes hyperboliques, seraient un avantage, mais ne sont pas un prérequis.