Ecole Doctorale Carnot-Pasteur

Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :

Jeux à champ moyen et théorie de Riccati dans le contrôle optimal des données échantillonnées

Intitulé en anglais du sujet de these proposé :

Mean-Field Games and Riccati Theory in Optimal Sampled-Data Control

Unité de recherche : IMB (UMR 5584, Université de Bourgogne & CNRS)

Nom, prénom et courriel du directeur (et co-directeur) de thèse :

Directeur de thèse : Thomas CHAMBRION (thomas.chambrion@u-bourgogne.fr)
Co-directeur de thèse : Fabio CAMILLI (fabio.camilli@unich.it)
Co-encadrant de thèse : Cristian MENDICO (cristian.mendico@u-bourgogne.fr)

Domaine scientifique principal de la thèse :

Contrôle optimal et équations aux dérivées partielles

Domaine scientifique secondaire de la thèse :

Analyse numérique et applications

Description du projet scientifique :

This PhD project explores the intersection of three key areas: Mean-Field Games (MFGs), Riccati theory, and sampled-data control. It aims to develop a comprehensive theoretical framework and novel algorithms for analyzing and solving optimal control problems in large-scale, interconnected systems with sampled-data information.

MFGs provide a powerful framework for studying large populations of interacting agents, where each agent's optimal decision depends on the aggregate behavior of the population. This approach simplifies the analysis of complex systems by approximating the individual agent's problem with a representative agent interacting with a mean-field representing the population's distribution. However, in many practical applications, control systems operate with sampled information, where measurements are available only at discrete time instants. This introduces

challenges in analysis and design, as the continuous-time dynamics of the system interact with the discrete-time nature of the sampled data.

This project aims to:

- **Develop a Mean-Field Game framework for sampled-data control systems:** Investigate how the mean-field approximation can be applied in the context of sampled-data information. This includes defining appropriate mean-field dynamics and exploring the convergence of the individual agent's optimal control to the mean-field solution.
- **Extend Riccati theory to sampled-data MFGs:** Derive Riccati-type equations for sampled-data MFGs, characterizing the optimal control laws and value functions. Investigate the impact of sampling frequency on the Riccati solutions and the overall system performance.
- **Develop numerical methods for solving sampled-data MFGs:** Design efficient numerical algorithms for solving the coupled system of mean-field equations and Riccati equations arising in sampled-data MFGs. This may involve exploring techniques from numerical optimization, dynamic programming, and model predictive control.
- **Investigate applications of sampled-data MFGs:** Explore potential applications of the developed theory and algorithms in various domains, such as networked control systems, multi-agent robotics, and traffic flow management.

References:

- **L. Bourdin, E. Trélat**, *Convergence in nonlinear optimal sampled-data control problems*, IEEE Trans. Automat. Control **69** (2024), no. 10, 7144--7151.
- **L. Bourdin, E. Trélat**, *Unified Riccati theory for optimal permanent and sampled-data control problems in finite and infinite time horizons*, SIAM J. Control Optim. **59**(2021), no. 2, 489--508.
- **L. Bourdin, E. Trélat**, *Linear-quadratic optimal sampled-data control problems: convergence result and Riccati theory,* Automatica Volume 79, 2017, Pages 273-281.
- **L. Bourdin, E. Trélat**, *Optimal sampled-data control, and generalizations on time scales*, Math. Control Relat. Fields **6** (2016), no. 1, 53--94.
- **L. Bourdin, E. Trélat**, *Pontryagin maximum principle for optimal sampled-data control problems*, Proc. CAO 2015, IFAC-PapersOnLine 48-25 (2015), 80--84.
- **P. Cardaliaguet.** Notes on mean field games from P. -L. Lions lectures at collège de France.
- **J.-M. Lasry and P.-L. Lions.** Jeux à champ moyen. I: Le cas stationnaire. *C. R., Math., Acad. Sci. Paris*, 343(9):619–625, 2006.
- **J.-M. Lasry and P.-L. Lions.** Jeux à champ moyen. II: Horizon fini et contrôle optimal. *C. R., Math., Acad. Sci. Paris*, 343(10):679–684, 2006.
- **J.-M. Lasry and P.-L. Lions.** Mean field games. *Jpn. J. Math. (3)*, 2(1):229–260, 2007.

Connaissances et compétences requises :

The ideal candidate should have a background in control theory and PDEs. Experience with differential games or numerical methods would be beneficial.

This PhD project offers a challenging and rewarding opportunity to contribute to the cutting-edge of research in control systems, game theory, and numerical methods. It will equip the candidate with advanced theoretical and computational skills, preparing them for a career in academia or industry.