École Doctorale Carnot-Pasteur

Proposition de sujet de thèse

Intitulé français du sujet de thèse proposé :
Méthodes homologiques et déformations dispersives des hiérarchies intégrables

Intitulé en anglais :
Homological methods and dispersive deformations of integrable hierarchies

Unité de recherche :
Institut de Mathématiques de Bourgogne (IMB)

Nom, prénom et courriel du directeur (et co-directeur) de thèse :
CARLET, Guido (guido.carlet@u-bourgogne.fr)

Domaine scientifique principal de la thèse :
Mathématique physique, systèmes intégrables.

Domaine scientifique secondaire de la thèse :
Mathématique physique, systèmes intégrables.

Description du projet scientifique

The theory of integrable systems has been the subject of intense research over the past forty years. Connections with many different fields have been established, one of the most fruitful being that with enumerative geometry and in particular with the theory of Gromov-Witten invariants [1], which has let to introduction of Frobenius manifolds [2], originally introduced as a way to encode the structure of 2D topological field theory.

In this framework - where Hamiltonian and bi-Hamiltonian structures appear as naturally associated to Frobenius manifolds and to their dispersionless (genus zero, hydrodynamic type) hierarchies of integrable equations [3] - the problem of dispersive deformation and classification of these structures was first formulated [1].

The field has developed rapidly [4–6], and recently the application of homological algebra techniques to the computation of Poisson and bi-Hamiltonian cohomology groups [7–9], has led to a complete description of the full cohomology in several relevant cases, including the proof of the conjecture [10] on the existence of general deformations of semi-simple bi-Hamiltonian structures of hydrodynamic type [11].

Recently similar techniques have been used [12–14] to solve long standing problems about the classification of the integrable hierarchies of topological type.

This project’s aim is to contribute the study of the existence and classification of dispersive deformations of structures in the theory of integrable hierarchies. Several open problems include the classification of Poisson structures with several independent variables, of bi-Hamiltonian structures, the application of cohomological methods to the quasi-Miura transformations and to the quasi-triviality of bi-Hamiltonian structures and hierarchies, to the existence of dispersive versions of the Virasoro and larger symmetry groups and the equivalence of the DZ and DR constructions of integrable hierarchies.

Connaissances et compétences requises :
The candidate should have a background in Mathematical Physics, either from a master in Mathematics (differential geometric, analytical mechanics and possibly algebraic methods) or in Physics (with a strong mathematical component).

Références

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3. Dubrovin, B., Novikov, S.P.: Hydrodynamics of weakly deformed soliton lattices. Differential geometry and Hamiltonian theory. Uspekhi Mat. Nauk. 44, 29–98, 203 (1989). https://doi.org/10.1070/RM1989v044n06ABEH002300
4. Getzler, E.: A Darboux theorem for Hamiltonian operators in the formal calculus of variations. Duke Math. J. 111, 535–560 (2002)
5. Dubrovin, B., Liu, S.-Q., Zhang, Y.: On Hamiltonian perturbations of hyperbolic systems of conservation laws. I. Quasi-triviality of bi-Hamiltonian perturbations. Commun. Pure Appl. Math. 59, 559–615 (2006)
6. Barakat, A., De Sole, A., Kac, V.G.: Poisson vertex algebras in the theory of Hamiltonian equations. Jpn. J. Math. 4, 141–252 (2009). https://doi.org/10.1007/s11537-009-0932-y
7. Carlet, G., Posthuma, H., Shadrin, S.: Bihamiltonian cohomology of KdV brackets. Commun. Math. Phys. 341, 805–819 (2016). https://doi.org/10.1007/s00220-015-2540-4
8. Carlet, G., Casati, M., Shadrin, S.: Poisson cohomology of scalar multidimensional Dubrovin–Novikov brackets. J. Geom. Phys. 114, 404–419 (2017). https://doi.org/10.1016/j.geomphys.2016.12.008
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12. Liu, S.-Q., Wang, Z., Zhang, Y.: Variational Bihamiltonian Cohomologies and Integrable Hierarchies I: Foundations. ArXiv210613038 Math-Ph. (2021)
13. Liu, S.-Q., Wang, Z., Zhang, Y.: Variational Bihamiltonian Cohomologies and Integrable Hierarchies II: Virasoro symmetries. ArXiv210901845 Math-Ph. (2021)
14. Liu, S.-Q., Wang, Z., Zhang, Y.: Linearization of Virasoro symmetries associated with semisimple Frobenius manifolds. ArXiv210901846 Math-Ph. (2021)