International Master in Mathematical Physics
– Math4Phys –

Guide 2026-27

Contents

1. Presentation

The Master in Mathematical Physics "Math4Phys" is a graduate program offered by the Department of Mathematics of the *Université Bourgogne Europe* (UBE), formerly *Université de Bourgogne*. The program is hosted at the *Institut de Mathématiques de Bourgogne* (IMB) in Dijon.

The program's primary objective is to provide advanced training in the mathematical methods of modern theoretical physics within a structured mathematical curriculum.

Such an offer exists in France only in Dijon, where the Mathematical Physics group at the IMB provides a unique environment for a program requiring expertise in both Mathematics and Physics.

The Mathematical Physics group of the IMB laboratory in Dijon is a distinguished research team in France with the ability to provide advanced lectures on the mathematical problems of modern physics.

2. Contacts

Program website: `math.ube.fr/math4phys`

General inquiries: `math4phys@ube.fr`

Address: Institut de Mathématiques de Bourgogne, UMR CNRS 5584, 9 avenue Alain Savary, BP 47870, 21078 Dijon Cedex, France

Secretariat:
Mylène MONGIN – office R24A – `secretariat.maths@ube.fr`

International Masters coordinator (International students support):
Eloïse ROUSSEL – `eloise.roussel@ube.fr`

Program coordinators:
Guido CARLET – M1 – office A415 – `guido.carlet@ube.fr`
José Luis JARAMILLO – M1 – office A323 – `jose-luis.jaramillo-martin@ube.fr`
Nikolai KITANINE – M2 – office A405 – `nikolai.kitanine@ube.fr`

3. Application, enrolment, and other useful information

For detailed information on application and enrollment procedures in English, please visit:
`www.ube.fr/en/international-forthem/international-students/obtaining-a-degree/`

All applications must be submitted exclusively via the eCandidat platform:
`ecandidat.ube.fr/`

4. Scholarships

Each year, the Master's program awards several scholarships of approximately 550 euros per month for up to nine months, based exclusively on academic merit. The number of scholarships depends on secured funding, which may vary annually. All students accepted in M1 and M2 are automatically considered for the scholarships, no separate application is required. Scholarship recipients must confirm their participation within two weeks of receiving the offer.

5. Statistics

Number of students	2021/22	22/23	23/24	24/25	25/26	26/27
who applied to M1	80	102	80	99	163	-
admitted to M1	32	34	35	44	41	-
registered to M1	14	14	12	14	13	-
registered to M2	14	14	14	13	23	-
who have completed M2	12	13	11	-	-	-
who have started a PhD after completing M2	7	11	6	-	-	-

6. Calendar 2026/27

The dates are provisional.

Applications begin	9/2/2026
Applications end	22/5
First meeting for M1 students	21/9 9:00
First meeting for M2 students	21/9 16:00
Toussaint *	25/10 – 3/11
Courses end	19/12
Noël	20/12/2026 – 5/1/2027
Exams	5/1 – 9/1
Courses begin	12/1
Hiver	14/2 – 23/2
Printemps	4/4 – 20/4
Courses end	7/5
Dissertation deadline	20/5
Exams, 1st session	11/5 – 18/5
Exams, 2nd session	15/6 – 26/6

* In *italics* the teaching breaks.

7. Courses M1

We propose nine main courses plus a language course and a dissertation. Each course (apart from the language course) consists of 22 hours of lectures (CM) and 22 hours of exercise classes (TD). For French-speaking students, the FLE course will be replaced by an English course.

	CM+TD (hrs)	ECTS
Groups and representations	22+22	7
Differential geometry	22+22	7
Functional analysis	22+22	7
Differential equations in the complex domain	22+22	7
FLE (or English)	0+20	2
Mathematical methods of classical mechanics*	22+22	6
Quantum mechanics for mathematicians*	22+22	6
Partial differential equations*	22+22	6
Computational methods in mathematical physics*	22+22	6
Statistical mechanics and stochastic processes*	22+22	6
Dissertation		6

* Students must choose 4 out of the 5 available courses.

The schedule of the courses will be accessible via the ENT website of the *Université Bourgogne Europe* at the address: `plannings.ube.fr/`

All courses are taught by members of the IMB in Dijon, unless otherwise noted. Other affiliations include the *Laboratoire Interdisciplinaire Carnot de Bourgogne* (ICB).

Groups and representations (P. Schauenburg, J. L. Jaramillo)

Notion of a group representation. Development of the structure theory for complex representations of finite groups: Theorems of Maschke and Schur. Tensor products and duality. Character theory. Induced representations. Some outlook beyond finite groups as time permits.

Differential geometry (R. Uribe-Vargas, M. Fairon)

Differentiable manifolds. Vector fields and flow-box theorem. Differential forms and Stokes' theorem. Tensors and vector bundles. Riemannian manifolds and connections. Geometry of gauge fields.

Functional analysis (G. Dito, C. Mendico)

1. Basic notion on topology. Compacts spaces, bounded linear maps on normed spaces. Compact and complete spaces. 2. Hilbert spaces. Projection theorem. Hilbertian orthonormal basis. Riesz representation theorem. Applications to Fourier series. Fejér theorem. 3. Linear operators on a Hilbert space. General theory of operators. Adjoint of an operator. Closed, self-adjoint, symmetric, unitary, normal operators. Application to differential operators. 4. Spectral theory. Basic concepts: point, continuous, and residual spectra. Resolvent operator. 5. Spectral theorem for compact operators. Applications to integral equations. Spectral theorem for bounded self-adjoint operators.

Differential equations in the complex domain (G. Carlet, N. Kitanine)

ODEs in the real domain: elementary methods, existence and uniqueness theorems (Cauchy-Peano, Picard-Lindelöf), linear systems. ODEs in the complex domain: scalar equations of the first and second order, systems, Riccati equation; complex linear systems, monodromy and singularities, Fuchsian systems, Bessel equation, hypergeometric equation; irregular singularities, Stokes matrices.

Mathematical methods of classical mechanics (G. Carlet, J. L. Jaramillo)

Lagrangian and Hamiltonian formalisms. Hamiltonian systems on symplectic manifolds. Variational principle and Hamilton-Jacobi equations. Poisson manifolds. Symmetries and momentum map.

Quantum mechanics for mathematicians (N. Kitanine, S. Carrozza)

1. Introduction: 1.1 Observables in classical mechanics 1.2. Finite dimensional model of quantum mechanics 2. Basic principles of quantum mechanics: 2.1 States and observables in quantum mechanics 2.2 Quantum entanglement 2.3 Heisenberg uncertainty principle 2.4 Coordinate and momentum representations 2.6 Quantum dynamics: Schrödinger and Heisenberg pictures 2.5 Schrödinger equation 2.6 Classical limit 3. Quantum mechanics in one dimension: 3.1 Harmonic oscillator. Creation and annihilation operators 3.2 Scattering problem in one dimension 4. Quantum mechanics in 3D: 4.1 Free particle 4.2 Rotation group and angular momentum 4.3 Hydrogen atom 4.4 Spin 5. Multi-particle quantum systems, introduction.

Partial differential equations (J. Lampart – ICB)

This course is an introduction to linear partial differential equations and relevant concepts from functional analysis. Tempered distributions; Fourier transform; Sobolev spaces; PDE with constant coefficients. Hilbert spaces; Bounded operators, linear functionals; Lax-Milgram Theorem and elliptic equations. Unbounded Operators; Self-adjoint and maximal dissipative operators; Evolution equations and the Hille-Yosida Theorem. Applications to the heat, wave and Schrödinger equations with potentials and variable coefficients.

Computational methods in mathematical physics (N. Stoilov)

Interpolation and/or Linear systems. Numerical integration (classical rules, Gaussian quadrature rules). Fourier approximation. Numerical methods for solving ODE and PDE.

Statistical mechanics and stochastic processes (J. L. Jaramillo, S. Fang)

Statistical mechanics: 1. Thermodynamics formalism. 2. Classical Statistical Mechanics and Ergodic Hypothesis. 3. Statistical Ensembles (and Large Deviation Theory). 4. Phase transitions, Ising model, renormalization group. 5. Non-equilibrium statistical mechanics: Onsager reciprocity and fluctuation-dissipation theorem. *Stochastic processes*: 6. Probability concepts. 7. Brownian motion and diffusion. 8. Ito calculus and stochastic differential equations. 9. The Fokker-Planck equation. 10. Detailed balance: Onsager relations and fluctuation-dissipation theorem (revisited).

8. Courses M2

There are eight main courses plus a course of languages and a dissertation. Three courses will focus on the specific theme of the year. For French-speaking students the course FLE will be replaced by a course of English.

	CM+TD (hrs)	ECTS
Lie groups and Lie algebras	18+18	7
Mathematical methods of quantum field theory	18+18	7
Riemann surfaces and integrable systems	18+18	7
Stochastic processes*	15+15	7
FLE (or English)	0+20	2
Path integral approach in QFT	15+15	5
General relativity	15+15	5
Random matrix theory and physics*	15+15	5
Random walk on graphs*	15+15	5
Dissertation		10

* Thematic courses.

The schedule of the courses will be accessible via the ENT website of the *Université Bourgogne Europe* at the address: `ent.u-bourgogne.fr`

All courses are taught by members of the IMB.

8.1. **Curricular courses.**

Lie groups and Lie algebras (G. Dito)

1. Lie algebras: Basic definitions. Ideals and Lie subalgebras. Lie theorems. Real and complex forms. Universal enveloping algebra. Poincaré-Birkhoff-Witt theorem. Campbell-Hausdorff formula. 2. Structure of Lie algebras: Solvable, nilpotent and semisimple Lie algebras. Killing form. Lie and Engel theorems. Cartan criterion. Jordan decomposition. 3. Semisimple Lie algebras: Cartan subalgebra.

Root system. Dynkin diagram. Classification of simple Lie algebras. Finite dimensional representations of sl(2).

Mathematical methods of quantum field theory (N. Kitanine)

1. Introduction, necessary background in mathematics and physics: distribution theory, functional analysis (spectral theorem), Lagrangian and Hamiltonian mechanics, basic principles of quantum mechanics, special relativity (Lorentz group and corresponding Lie algebra). 2. Classical field theory: Lagrangian formulation, conservation laws, Noether theorem for fields, examples (Klein-Gordon equation, sine-Gordon equation, non-linear Schrodinger equation, Maxwell equations etc.). Hamiltonian formulation, Poisson brackets. 3. Canonical quantisation of the free bosonic field: Weyl algebra, Fock space for bosons, lattice bosons, finite volume, infinite volume limit. Free bosons in 3+1 d. 4. Spinor representation of the Lorentz group and Dirac equation. Canonical quantization: Clifford algebra, Fock space for fermions: lattice fermions, finite volume, infinite volume limit. Free fermions in 3+1 d. 5. Introduction to the interacting field theories: Interaction picture, Dyson formula, Wick theorem, Feynman diagrams. 6. (if there is time) Introduction to quantum integrability.

Riemann surfaces and integrable systems (G. Carlet)

Definition of Riemann surface and basic examples, plane algebraic curves, hyperelliptic curves, holomorphic coverings, fundamental group, Riemann-Hurwitz theorem, homology groups, sheaves, Cech and sheaf cohomology, meromorphic functions, abelian differentials, integration theorems, divisors, Abel-Jacobi map, Abel theorem, Riemann-Roch theorem, Serre duality.

Path integral approach in QFT (T. Kimura)

Lagrangian formalism and symmetry. Path integral formalism. Interacting fields and perturbation theory. Loop correction and renormalization. Quantization of non-Abelian gauge theory. Spontaneous symmetry breaking.

General relativity (S. Carrozza, B. Raffaelli)

The bulk of the course will focus on basic aspects of General Relativity. Outline: 1. Minkowski spacetime: special relativity, proper time, metric, causal structure and conformal compactification, energy-momentum tensor. 2. Geometry of curved spacetimes: manifolds, tensor fields, Lie derivative, covariant derivatives, Levi-Civita connection, curvature, geodesics. 3. Einstein's field equations: (heuristic) derivation, Lovelock theorem, linearized gravity, Newtonian limit. 4. Particular solutions of Einstein's equations: Schwarzschild black hole, cosmological solutions, and their singularities. Time-permitting, we will explore the concept of spacetime singularity in more depth, by reviewing more advanced notions such as: congruences of curves, the Raychaudhuri equation, energy conditions, and singularity theorems.

8.2. **Thematic courses.**

Each year the master proposes thematic courses on a specific theme.

Year	Theme
2024-25	Introduction to quantum topology
2025-26	Algebraic geometry
2026-27	Probabilistic analysis in mathematical physics
2027-28	Classical integrable systems

See Appendix for the thematic courses of the past years.

8.2.1. *Thematic courses 2026-27.*

For the year 2026-27 the theme is “Probabilistic analysis in mathematical physics”.

Stochastic processes (S. Herrmann)

This is a second-level course on stochastic processes. A number of basic notions have already been introduced in the first year of the Master’s program (”Statistical Mechanics and Stochastic Processes”) and will form the building blocks of this advanced course. In particular, the notion of diffusion and the link with the Fokker-Planck equation are introduced in the first year. This advanced course in probability theory offers another approach essentially based on the theory of martingales.

1. In a first part, we introduce the notion of conditioning and develop the theory of martingales, taking care to illustrate the theoretical results with examples of stochastic processes. The discrete time framework is first addressed and then the continuous time with the construction of Itô’s integral. The main properties that make martingales indispensable will be discussed: Doob-Meyer decomposition, optimal stopping theorem, convergence in large time, etc.
2. Secondly, we will examine the link between the mean behaviour of specific families of stochastic processes and solutions of PDEs (elliptic or parabolic type equations). This will allow us to study, in particular, initial value problems for classical physics equations. We will take this opportunity to describe the phenomena of bistability and metastability, which are all linked to the escape problems of stochastic processes.
3. Asymptotic analysis plays an important role when physical systems involve stochastic processes and will be the main theme of this last part of the course. The first case concerns small noisy perturbations of dynamical systems. In this case, it is necessary to be able to control the limiting behaviour when the noise intensity becomes small, using the principle of large deviations (Sanov, Schilder, Freidlin-Ventzell). The second case concerns the time dependence of noisy systems that admit a stationary regime: we are then interested in the ergodic theorems that describe the behaviour in large time.

Random matrix theory and physics (S. Carrozza)

This course will provide an introduction to Random Matrix Theory (RMT) [1, 2, 3], with an emphasis on some of its numerous applications to physics. RMT entered the world of physics through Wigner, who had the insight to model the Hamiltonian of a quantum chaotic system as a large random Hermitian matrix, which allowed him to exploit concentration phenomena to investigate its spectral properties. We will start out by introducing basic ensembles of random matrices (Wigner and Wishart matrices, Gaussian ensembles) and will investigate the limit distributions of their eigenvalues in the asymptotic regime of large dimension (semi-circle law, Marchenko-Pastur distribution). We will show that those ensembles are subject to the phenomenon of eigenvalue repulsion, which plays an important role in Wigner’s analysis of quantum chaotic systems. Still in the context of quantum chaos, we will then discuss a threefold classification of random Hamiltonians in terms of their symmetries (Wigner’s threefold way), as well as a tenfold generalization due to Altland-Zirnbauer [4] (which is in correspondence with the classificaltion of associative division super-algebras). To conclude this part, we will discuss some applications of RMT to quantum information [5], such as the computation of the typical entanglement spectrum of a random bipartite pure state (whose entanglement entropy is captured by the celebrated Page curve). In the second part of the course, we will focus on matrix models, which can be interpreted as formal non-Gaussian random matrix distributions (or, in theoretical physics language, as perturbative matrix field theories in zero dimen- sion). We will explain how the (formal) moments of such a distribution can be understood as generating functions of objects known as combinatorial maps (or ribbon diagrams), which are nothing but discrete surfaces [6, 7]. We will then explain how this expansion can be re-organized as a so-called topological expansion in which the genus of a given discrete surface determines the order of its contribution in the (formal) parameter N1 , N being the size of the matrix. After briefly discussing the original context in which this expansion was discovered by 't Hooft (large N quantum field theory, and more specifically SU(N) quantum chromodynamics), we will give a more detailed account of its applications to combinatorics and statistical physics [8]. From a purely combinatorial perspective, the large N expansion allows to solve asymptotic enumeration problems for combinatorial maps. From a more physical perspec- tive, the same approach allows to generate and solve statistical physics models on random surfaces. We will discuss two such examples: a random surface model which can be understood as pure Euclidean 2d quantum gravity; and the Ising model on a random surface. In the third part of the course, we will provide an introduction to free probability. This is an operator-algebraic formalism which fits in the general framework of noncommutative probability, and which relies on a particular gen- eralization of the classical notion of independence known as free independence. Random matrices turn out to be asymptotically free, which means that their asymptotic properties are well-captured by the abstract framework of free probability. The tools of free probability are particularly useful to generate and describe more complicated RMT ensembles than the ones studied in the first part of the course. We will adopt an approach to free probability rooted in combinatorics [9], and illustrate the power of this formalism with concrete examples drawn from mathematical physics. Finally, time-permitting, we will provide a quick overview of the emerging theory of random tensors, which gener- alizes aspects of RMT discussed in this course to higher order tensors [10].

Random walk on graphs (A. Rousselle)

Random walks on graphs are widely used in all sciences to describe a great variety of phenomena where dynamical random processes are affected by topology and represents at the same time a basic model of diffusion phenomena and nondeterministic motion. This lecture presents general random walks on (weighted, finite or infinite, deterministic or random) graphs. This teaching draws its main ideas from physical phenomena, since it links random walks to electrical networks by interpreting the return probabilities for the walk as potential for the associated electrical networks. In particular, the characterization of the recurrence/transience property of a graph can be intepreted in terms of resistance to infinity. This enables us to give robust criteria for recurrence/transience on infinite graphs.

1. In this course, we will first concentrate on random walks on deterministic graphs. This will enable us to define the various mathematical tools used in a fairly simple context, in particular recurrence and transience. An important result concerning random walks which is called the classical invariance principle asserts that the distributions of a broad class of continuous functionals of processes constructed from a rescaled random walk converge to the distributions of these functionals of a Brownian motion.
2. Secondly, we introduce more general models by considering random environments (random walks on random graphs). Physicists have been mainly interested in graphs as models of complex systems. Indeed, these struc- tures are very useful to describe inhomogeneous structures such as disordered materials, glasses, polymers, biomolecules as well as electric circuits, communication networks, statistical models of algorithms, and applica- tions of statistical mechanics to different (non physical) systems. Therefore making the graph random broadens the scope of the mathematical results. We address the concepts of almost sure reccurence and transience, annealed and quenched invariance principles,...
3. Finally we introduce the so-called Maximal Entropy Random Walk (MERW) whose transition probabilities have been chosen to maximise entropy. The properties of this biased random walk make it particularly useful in analysis of complex networks. We begin by studying the case of a finite connected graph and continue with general MERW, MERW with energy constrains,... Asymptotic results (principles of invariance and large deviations) are also discussed.

8.2.2. *Thematic courses 2027-28.*

For the year 2027-28 the theme is "Classical integrable systems".

Finite Dimensional Integrable Systems (T. Combot and C. Mendico)

Consider a vector field X on a finite dimensional manifold, and the solutions of the differential system $\dot{x} = X(x)$. When the solutions are not exponentially sensitive to the initial conditions, we want to call such system integrable. The course will be divided in two parts.

1. During the first part, we will introduce formal definitions of integrability, especially in the case of Hamiltonian systems. We will prove that such integrable systems have quasi periodic dynamics on a foliation of Lagrangian tori, and that the solutions can be explicitly constructed. These notions will be concretely applied to various examples, notably classical integrable cases, the pendulum, tops, geodesic flows and the two body problem. We will also present obstructions to obtaining global explicit formulas for the solutions, in particular due to monodromy.
2. The second part is devoted to the analysis of invariant structures for integrable and non-integrable Hamiltonian systems. In particular, we will address the proof of the KAM theorem concerning the persistence of so-called invariant tori, as well as the weak KAM theorem using viscosity solutions to ergodic Hamilton-Jacobi equations. Regarding the latter, we will consider both the variational approach (Aubry-Mather theory) and the PDE approach (weak KAM theory), concluding this second part with an analysis of the singularities associated with invariant sets.

Although the course will rely heavily on the study of explicit examples, basic notions of differential geometry and differential equations as provided in the M1 will be needed.

References:

- M. Audin, Hamiltonian systems and their integrability. American Mathematical Soc.,2008.
- R.M. Cushman, and L.M. Bates. Global aspects of classical integrable systems. Vol. 94. Basel: Birkhäuser, 1997.
- A.V. Bolsinov, and A.T. Fomenko. Integrable Hamiltonian systems: geometry, topology, classification. CRC press, 2004.
- V.I. Arnold et al. Mathematical aspects of classical and celestial mechanics. Vol. 3. Berlin: Springer, 2006.
- A. Fathi. Weak KAM Theorem and Lagrangian Dynamics. unpublished, 2008.
- A. Sorrentino, Action-minimizing methods in Hamiltonian dynamics. An introduction to Aubry-Mather theory. Mathematical Notes (Princeton) 50. Princeton, NJ: Princeton University Press (ISBN 978-0-691-16450- 2/pbk; 978-1-400-86661-8/ebook). xi, 115 p. (2015).
- Notes on Hamiltonian dynamical systems, London Mathematical Society Student Texts 102.

Infinite dimensional integrable systems (M. Fairon and N. Stoilov)

In the first part of this course, we will examine the notion of integrability from a symmetry point of view. Starting with the Painlevé property for ordinary differential equations, we will move on to define point, contact and higher symmetries. Then, we will discuss integrable partial differential equations, the notion of symmetries, Lax pairs and the important construction of soliton solutions through an inverse scattering problem. We will study in detail the Korteweg-de Vries, the nonlinear Schrödinger, and some other equations and their physical applications.

In the second part of this course, we will develop a powerful algebraic formalism for studying infinite-dimensional Hamiltonian systems based on Poisson vertex algebras. The construction of integrable hierarchies of partial differential equations will be introduced in this context. Furthermore, the relation to the classical theory of Hamiltonian operators will be described, and the Lenard-Magri scheme of integrability will be presented.

References:

- M.J. Ablowitz and P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, Cambridge, 1991
- M. Dunajski, Solitons, instantons, and twistors. Oxford University Press, 2010
- V. Kac, Introduction to Vertex Algebras, Poisson Vertex Algebras, and Integrable Hamiltonian PDE. In: Perspectives in Lie Theory, Springer INdAM Series, vol. 19, Springer, 2017; pp.3-72
- P.J. Olver, Applications of Lie Groups to Differential Equations, Graduate Texts in Mathematics, vol. 107, Springer, New York, 1993

Algebro-Geometric solutions (C. Klein)

The search for periodic solutions of completely integrable PDEs has led to remark- able progress both in the theory of integrable systems and in algebraic geometry. In this course we will explore quasi-periodic solutions via the so-called Baker-Akhiezer function, a generalization of the exponential function to Riemann surfaces. Concrete examples as the Korteweg-de Vries and the Kadomtsev-Petviashvili equations will be studied. The solution to the latter provides an answer to the classical Schottky problem which symmetric matrices with positive definite imaginary part are period matrices of a Riemann surface.

An alternative way to algebro-geometric solutions is provided by Fay?s celebrated trisecant identity for secants in the Kummer variety and suitable degenerations thereof, an approach developed by Mumford. The pinching of cycles on the underlying Riemann surfaces leads to solitonic solutions. For the example of the Ernst equation, which is equivalent to the stationary axisymmetric Einstein equations in vacuum, this leads to black hole spacetimes which are thus systematically derived.

References:

- E.D. Belokolos, A.I. Bobenko, V.Z. Enolskii, A.R. Its, V.B. Matveev, Algebro-Geometric Approach to Nonlinear Integrable Equations, Springer, Berlin, 1994
- B.A. Dubrovin, Theta functions and non-linear equations, Russian Math. Surveys 36 (1981) 11
- A. Bobenko and C. Klein (ed.), Computational Approach to Riemann Surfaces, Lecture Notes in Mathematics Vol. 2013 (Springer) (2011)
- C. Klein and J.-C. Saut, Nonlinear dispersive equations ? Inverse Scattering and PDE methods, Applied Mathematical Sciences 209 (Springer, 2022)
- D. Mumford, Tata lectures on theta II. Progress in Mathematics, vol. 43. Boston, MA: Birkhäuser Boston, Inc. (1984)
- C. Klein and O. Richter, Ernst Equation and Riemann Surfaces, Lecture Notes in Physics Vol. 685 (Springer) (2005)

9. Dissertation M1

The students are required to choose a supervisor and a topic in October and to work on the project under their guidance throughout academic year. Depending on the number of students, the project may need to be completed in pairs.

The completed work must be summarized in a dissertation, which should be submitted by May.

The general rules for the dissertation are the following:

1. a maximum of 25 pages in TeX,
2. it should not be a direct rephrasing of some chapter of a book,
3. it should contain some personal and detailed take on a specific proof or computation.

Extra material can be included in the appendix without limitation on the number of pages, but won't be evaluated.

The dissertation must be submitted by the deadline to one of the coordinators and will be evaluated by the supervisor.

The guidelines for evaluation are the following: understanding of the material, quality of the writing, originality in the treatment of the topic, engagement of the student.

The assessment of the supervisor takes into account the work done by the student during the year and the quality of the dissertation.

All dissertations will be checked using plagiarism detection software. In the case of evident plagiarism the mark will be zero.

It is not possible to retake the evaluation of the dissertation in the second session.

10. Dissertation M2

The students are required to choose a supervisor and a topic during the month of October and to work on the project under the guidance of the supervisor during the whole academic year. For more details, please contact the M2 coordinator.

11. Exams

The exams of the first semester take place in January, those of the second semester in May, see the calendar above. Exams are typically written tests lasting two to three hours.

Possible outcomes of an exam are a mark between 0 and 20 or *DEF* (*défaillant*). A course is passed if the mark is greater or equal than 10. *DEF* is attributed to a student that is absent at the final exam, or to a student that is absent at the partial exam and does not have an official justification. Students that are absent at the partial exam but have an official justification will be given a zero mark, unless an extra session is organised, at the discretion of the teacher.

If a student has one or more exam scores below 10 during a semester but achieves an overall average of 10 or higher, the lower scores will be validated under the "compensation" system. Exams are also validated by compensation if the average of the marks of the year is greater or equal to 10. The marks of the exams validated by compensation cannot be improved by retaking the exams later. If an exam is marked *DEF* then no course in that semester can be validated by compensation.

Exams that are not passed at the first attempt and are not validated by compensation can be retaken in the second session in June.

The general rules for the exams are detailed in the official document "Référentiel commun des études – Université Bourgogne Europe", available here:
`formations.ube.fr/fr/organisation-des-etudes.html`

12. Repeating the M1

Because the Master receives several outstanding applications each year and the number of available places is limited, students that fail the first year in general will not be allowed to repeat it. Only exceptional circumstances will be considered.

Students that have failed the second session of exams in June and wish to repeat the year should address their application to the coordinators within one week of the publication of the results of the second session.

The students readmitted to the M1 will not benefit from a scholarship.

13. F.A.Q.

I have a mixed background in mathematics/physics/other subject. Do I have any chance to be accepted in the Master?

As long as you have a background in mathematics or physics, even if you studied other topics, you have a chance to be selected. We cannot say anything in advance however, the only way to know is to apply and let the admission committee examine your dossier.

How many recommendation letters should I attach to my application? Are they compulsory?

Recommendation letters are not required, but they can be attached to the application or sent to `math4phys@ube.fr` if one wishes so.

I have been accepted to the master, what should I do?

You will receive instructions from the administration about the procedure to register and other practical matters.

When should I confirm my participation to the master?

We suggest you confirm your participation to (or your withdrawal from) the master as soon as possible, to quickly advance the registration procedures. The formal deadline is in the beginning of September. If you are offered a scholarship you should confirm your participation within two weeks of the offer.

At which stage can I apply for a scholarship?

You will not need to apply specifically for a scholarship, if you are accepted at the Master you will be automatically considered for the scholarship.

How many students will be funded?

The number of available scholarships depends on annual funding, which may vary.

What are the requirements for the scholarships?

The selection is based on academic merit, no other criteria are considered.

What is included in a scholarship?

The scholarships are around 550 euros per month. Tuition fees are not included.

What is an estimate of the cost of life for a student in Dijon?

As an estimate, lodging is around 200 euros/month. Tuition fee is around 240 euros/yr and there is an extra compulsory contribution for 'student life' of 95 euros/yr.

When will scholarship recipients be announced?

Depending on the type of scholarship (we have different sources of financing), recipients will be announced in the months of July-August-September.

Where can I find other sources of scholarships?

Unfortunately we are not aware of other sources of funding. We advise international students to enquire at their own institutions for exchanges programmes with France or other funding to study abroad.

Is it possible to work part-time while following the master?

Due to the intensive course schedule, we do not recommend working while enrolled in the program.

Do I have to propose a subject for the dissertation or do I have to choose it from a list?

Usually several topics are proposed by the supervisors by the end of September. We encourage the students to discuss the topics with the supervisors and finalise their choice during the month of October.

I would like to have some material to study during the summer to be more prepared for the beginning of the semester.
During the summer a list of topics and relevant study materials will be sent to students accepted at the M1 and that have confirmed their participation.

Appendix A. Past thematic courses

We collect here the description of the courses provided in the framework of the past thematic years.

A.1. **Thematic courses 2024-25.**

For the year 2024-25 the theme was "Introduction to quantum topology". We proposed three courses. The first course gives the mathematical background for the object of study in quantum topology, i.e. low-dimensional manifolds (mainly 3-manifolds, including knots and links). The second course is an introduction to quantum algebra, and presents the algebraic structures underlying the construction of quantum invariants. The last and third course will provide examples of quantum invariants of 3-manifolds, and will consider their extensions to TQFTs ("Topological Quantum Field Theories").

An introduction to low-dimensional topology (G. Massuyeau)

Low-dimensional topology is characterized by its object of study, namely manifolds of "low" dimensions (no more than 4). In this introductory course, we will start with a short review of the classification of surfaces. Next, we will give an introduction to knots, links and tangles, which constitute special classes of 3-manifolds. In particular, we will define and study braid groups. Finally, we will see how to present arbitrary 3-manifolds via handle decompositions and surgery techniques. Although our approach will be mostly based on the study of examples, we shall also need some basic tools of algebraic topology and/or differential topology.

Tensor categories and quantum groups (P. Schauenberg)

Tensor categories and quantum groups are at the heart of a plethora of applications of algebraic methods to mathematical physics and topology often summarized under the moniker quantum algebra. Key words include topological quantum field theory, conformal field theory, the theory of quantum computation, integrable systems, deformation quantization, and noncommutative geometry. The lecture will give an introduction to the basic notions in the theory of tensor categories and important additional structures such as braidings and duality, an introduction to quantum groups (Hopf algebras) whose representation theory gives important examples of such structures, and discuss important constructions, notably the Drinfeld center/double and quantized enveloping algebras.

An introduction to topological quantum field theories (L. Woike, R. Detcherry)

The notion of a topological quantum field theory axiomatizes a certain type of quantum field theory for which the quantities associated to a region of spacetime only depend on the 'shape' of the spacetime. In addition to their physical origin, topological quantum field theories are also a subject of purely mathematical interest, and this course will be focused on the mathematical aspects. In the first half of the course, we will introduce the general axiomatics of topological quantum field theories compactly as symmetric monoidal functors from the cobordism category to the category of vector spaces. Then we will focus on the classification of two-dimensional topological field theories by commutative Frobenius algebras. In the second half, we build a class of examples of three-dimensional topological quantum field theories, the so-called Dijkgraaf–Witten theories, via the finite path integral quantization of a discrete gauge theory. As an outlook, we will discuss aspects of the Reshetikhin–Turaev construction of topological field theories from modular fusion categories.

A.2. **Thematic courses 2025-26.**

For the year 2025-26 the theme was "Algebraic geometry".

Introduction to algebraic geometry (L. Moser-Jauslin, Keyao Peng)

In the first part of the course will give an introduction to the basic concepts of complex algebraic geometry by studying affine and projective varieties. We will discuss the correspondence between varieties and their rings of functions. We will prove Hilbert's Nullstellensatz, study its consequences and use commutative algebra to study varieties. We will introduce the Zariski topology, and study its relationship to the usual topology of complex varieties. In particular, we will consider geometric and topological properties of algebraic varieties such as smoothness and tangent spaces. We will then concentrate on the study of plane curves, and the relation between these objects and Riemann surfaces.

We will also focus on methods of studying algebraic varieties using sheaves and categories. Students will explore the foundational concepts of algebraic geometry through the use of sheaf theory and its applications in differential geometry, aiming to bridge the gap between abstract algebraic concepts and their geometric interpretations, preparing students for advanced studies in mathematical physics.

Recommended references:

[1] "Complex Algebraic Curves", by Frances Kirwan, Cambridge Univ. Press, LMS Student texts 23
[2] "An Algebraic Geometry Primer for physicists", by Taizan Watari, `https://member.ipmu.jp/taizan.watari/Lectures/16a-KIPMU-ag4string/19a-KIMPU-ag4string-21may.pdf`

Cohomology of algebraic varieties (J. Nagel, M. Cavicchi)

Given a complex, smooth projective algebraic variety X, one can study it through various invariants. The singular cohomology groups are defined using the natural topology of X, the de Rham cohomology groups are computed in terms of differential forms, and the special nature of X as a complex manifold determines an additional datum on its cohomology, called a Hodge structure. The aim of this course is to discuss these topics, with an eye towards areas of mathematical physics where such invariants play a crucial role.

In a first part, we will define and study the aforementioned cohomology theories, introducing the homological tools that we need along the way. In the case of smooth projective curves, we will build the connection with the course on Riemann surfaces and we will prove by hand the Hodge decomposition theorem. In a second part, we will concentrate on Hodge theory in higher dimensions, with the final aim of computing the Hodge numbers of the quintic threefold appearing in mirror symmetry (a three-dimensional Calabi-Yau complex manifold). Time permitting, we will elucidate the connections with enumerative invariants, and/or give a glimpse of the construction of the mirror quintic.

Recommended references:

[1] "Algebraic geometry over the complex numbers", by D. Arapura, Universitext, Springer, New York, 2012
[2] "Differential forms in algebraic topology", by R. Bott, L. Tu, Graduate Texts in Mathematics 82, Springer-Verlag, New York-Berlin, 1982
[3] "Principles of algebraic geometry", by P. Griffiths, J. Harris, Wiley Classics Library, John Wiley & Sons Inc., New York, 1994

[4] "Hodge theory and complex algebraic geometry vol. I", by C. Voisin, Cambridge Studies in Advanced Mathematics 76, Cambridge University Press, Cambridge, 2007

Introduction to Hilbert schemes and moduli spaces (D. Faenzi)

This course is intended as an introduction to moduli spaces in algebraic geometry, through the guiding example of Hilbert schemes of points on a surface. We will follow closely the lecture notes [Nak99].

1. Moduli functors, Hilbert schemes. We will introduce the functorial approach to moduli problems, specify it to the case of Hilbert schemes and review some basic results on Hilbert schemes of n points on a smooth algebraic surface. We will see that this is a smooth and irreducible variety of dimension $2n$, that admits a natural birational morphism to the symmetric n-th power of the surface, which is actually a resolution of singularities. Useful material can be found in [Leh04].
2. Higher rank moduli spaces. This part is devoted to the study of moduli spaces of framed sheaves on the projective plane and the associated ADHM data. This relies on the concept of moduli space of vector bundles with a notion of stability, which in this case will be played by a framing along a line at infinity. We will see how to describe such moduli spaces explicitly via the Beilinson spectral sequence and the so-called ADHM data, which is to say a set of linear maps satisfying commuting relations and a stability condition, which are tantamount to a representation of a certain quiver.
3. Hyper-Kähler and symplectic structure. Here we will review how Hilbert schemes of points on the affine plane inherit a structure of an affine Hyper-Kähler manifold and how this is related to moment maps. On one hand, we will review basic material on Geometric Invariant Theory and on the other hand we will develop a bit the theory of symplectic quotients to show that the moduli spaces obtained by GIT in the previous chapter can actually be described as Hyper-Kahler quotiens.
4. Betti numbers. If time allows, we will get a glimpse of the co-called instanton counting, which is to say, computing the Betti numbers of the moduli spaces seen so far, notably the Hilbert scheme of points on a surface, and study how these numbers obey interesting combinatorial formulas related to partition functions.

Recommended references:

[Leh04] Manfred Lehn, Lectures on Hilbert schemes, Algebraic structures and moduli spaces, CRM Proc. Lecture Notes, vol. 38, Amer. Math. Soc., Providence, RI, 2004, pp. 1–30. MR 2095898

[Nak99] Hiraku Nakajima, Lectures on Hilbert schemes of points on surfaces, University Lecture Series, vol. 18, American Mathematical Society, Providence, RI, 1999. MR 1711344